From Fig. 3, the following conclusions can be drawn relative to the recessed wall flame holder operating with lean mixtures:

1) For a given mean velocity, the effect of boundary-layer removal on blowoff is to cause blowoff to occur at a higher equivalence ratio than the value at which blowoff occurs without suction.

2) For a given mean velocity, the blowoff equivalence ratio increases with suction rate to a maximum value and then decreases.

3) For a given mean velocity, the maximum equivalence ratio occurs at a suction rate less than that required for complete boundary-layer removal. The maximum equivalence ratio occurred at a suction rate of 70 to 75% of the boundary-layer flow.

4) The effect of boundary-layer removal on stabilization decreases at higher velocities.

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Self-Preserving Fluctuations and Scales for the Hypersonic Turbulent Wake

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Introduction

If the electric field incident upon each point of the underdense turbulent wake is assumed to be unperturbed by the scattered electric field (an assumption that has been referred to as a Born approximation), a knowledge of the statistical properties of the wake suffices to enable an approximate calculation of radar return.[1–4] Required is the double correlation function †$Q = \overline{\delta\eta(\mathbf{x})\delta\eta(\mathbf{x}+\mathbf{r})}$ of the passive field of dielectric constant fluctuations $\delta\eta$ at all points in the wake. This (scalar) quantity is generally a function of the vector position quantities $\mathbf{x}$ (measured, e.g., from the field source as origin) and $\mathbf{r}$ (measured from the point $\mathbf{x}$). For a homogeneous isotropic field, only the scalar polar radius r enters as argument. Alternatively, the (scalar) three-dimensional Fourier transform $E(\mathbf{k})$ of the correlation function may by specified; for an isotropic field, it is a function of the (scalar) wave number magnitude k. Because of the spherical symmetry of the isotropic field, it is sometimes useful to define a quantity $\hat{E}(k) = 4\pi k^2 E(k)$, which may then be interpreted as the density per unit thickness of $E(k)$ on a spherical shell of radius k in wave number space, i.e., as the line density along coordinate k.[5] As an example of the functional form that E may assume, consider the interpolation formula suggested by Hinze[6] for an isotropic scalar field (in analogy to a suggestion by von Kármán for velocity fluctuations):

$$\hat{E}(k) = 0.8\eta'^2\Lambda(k\Lambda)^2[1 + (k\Lambda)^2]^{-11/6} \tag{1}$$

where Λ is approximately the integral scale

$$\Lambda \approx \eta'^{-2}\int_0^\infty Q(r)dr$$

and η' the rms fluctuation level of η. This expression exhibits the proper k^2 behavior of an isotropic scalar field for small k and the Kolmogoroff $k^{-5/3}$ behavior for large k; the latter is appropriate‡ over a range of wave numbers for high Reynolds number (e.g., see Ref. 7). The use of a typical relation as the foregoing reduces the calculational problem to a determination of the variation of η' and Λ in the wake. In this note, the consequences of the assumption of a self-preserving fluctuation field will be examined for the fluid mechanical variables of a hypersonic wake. This useful assumption enables the fluctuating field to be calculated in terms of mean flow properties and provides a model that is expected to be valid at large distance from the body. However, at the present time, only rough inferences[8] may be made concerning the dielectric constant fluctuation and its relation to the fluid mechanical fluctuations; this important connection is beyond the scope of the present note.

Self-Preserving Flow

The concept of a self-preserving turbulent field is based on the presumption that the time required for the mean flow to vary, τ_D, is longer than a characteristic time for "adjustment" of the fluctuating field to a change in the mean flow τ_a. An often observed phenomenon in a turbulent field is that the time scale for "energy containing" eddies to interact and transfer their energy to higher wave numbers is of the order of the period of these eddies.[5] The energy containing eddies have a period of $\approx k_1 y_f/(U_0 - U_f)$, where $0.1 < k_1 < 1$. The time required for the mean flow to vary is y_f^2/ϵ. Taking $\epsilon = K(U_0 - U_f)y_f$ then the ratio of these times is $\tau_a/\tau_D = Kk_1$. Since $10^{-2} < K < 10^{-1}$,[9,10] τ_a/τ_D is smaller than unity, suggesting self-preservation for the energy containing eddies.

The detailed results of Townsend for an incompressible cylinder wake indicate that the far wake is indeed accurately self-preserving.[9] Whether a similar result will hold for a compressible axisymmetric wake remains to be shown. As in the case of the incompressible wake, a double structure is likely, i.e., small-scale high-intensity eddies embedded in slowly interacting low wave number eddies. The foregoing estimates, which apply to either compressible or incompressible flow, suggest that the large eddies do not maintain their identity very far downstream.§ In fact, the model of Townsend is that of local equilibrium of large eddies; these eddies "are not permanent structures, new ones arising as old ones disappear" (Ref. 6, p. 440). Further, their energy content, although not negligible, has been found to be small (less than 20% of total[9]). Thus, if one is concerned with the underdense far wake, local self-preservation may be reasonable as a rough approximation. It is at least interesting to examine the consequences of this assumption for typical hypersonic wakes of interest.

Fluctuation Levels

If the velocity fluctuation field is self-preserving, then by definition $(U'/U)^2 = a[(U_f - U)/U)]^2$, where a is a constant. Since $U \approx U_f$ and the stagnation enthalpy is con-

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† Bars over quantities refer to time or ensemble averaged properties. Primes refer to rms values. Subscripts 0 and f denote values on wake axis and at wake edge.

‡ The Reynolds number may not be sufficiently high for cases of interest.[8]

§ Estimates of the effect of finite time-constant mixing have recently been made by Proudian and Feldman.[11] A comparison of limiting cases has been carried out by Lin and Hayes.[12]

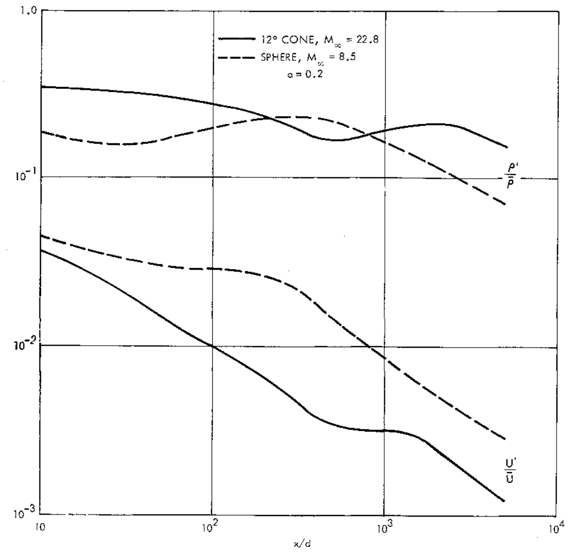

Fig. 1 Self-preserving fluctuations for slender cone and sphere wakes.

stant for most of the wake,[10] the foregoing may be written for the axis as $(U'/U)_0 = a^{1/2}h_\infty B(x)/U_0^2$, where $B(x) = (h_0 - h_f)/h_\infty$ as in the notation of Ref. 10. For fluctuation levels small relative to mean levels,¶ the linearized relations $(\rho'/\rho)_0 = (h'/h)_0$ and $(h'/h)_0 = (U_0^2/h_0)(U'/U)_0$ are obtained from the energy equation and equation of state upon ignoring pressure fluctuations and dissociation energy.** Combining the foregoing,

$$(\rho'/\rho)_0 = a^{1/2}(1 - h_f/h_0) \qquad (2)$$

Notice that $(\rho'/\rho)_{0\,\max} = a^{1/2}$ and that $(\rho'/\rho)_0 \to 0$ as $h_0/h_f \to 1$. Further, note that, if the gas density fluctuations are directly assumed to be self-preserving rather than the velocity, i.e., if we take

$$\left(\frac{\rho'}{\rho}\right)^2 = a\left(\frac{\rho_f - \rho}{\rho_f}\right)^2$$

the same result (2) is obtained (the constant a is unknown in either case).

Turbulent Scales

For a self-preserving flow, length scales measuring the energetic eddies vary as the dimension of the turbulent region. Thus, if θ is a characteristic dimension for the wake of an axisymmetric body, the incompressible or transformed scale Λ_0 should vary as $\Lambda_0/\theta = b(x/\theta)^{1/3}$, where b is a constant. The momentum defect thickness is

$$\frac{\theta_m^2}{d^2} = \frac{2}{\rho_f U_f^2}\int_0^{y_f/d} \rho U(U_f - U)\frac{y}{d}\,d\left(\frac{y}{d}\right) = \frac{C_{D_f}}{8}\frac{h_f}{h_\infty}$$

whereas the enthalpy excess thickness is

$$\frac{\theta_h^2}{d^2} = \frac{2}{\rho_f U_f h_f}\int_0^{y_f/d} \rho U(h - h_f)\frac{y}{d}\,d\left(\frac{y}{d}\right) = \frac{C_{D_f}}{8}(\gamma_\infty - 1)M_\infty^2$$

where C_{D_f} is the "local" drag coefficient of the wake,[10] and all dimensions have been normalized by body diameter d. The ratio of these thicknesses is

$$\theta_h/\theta_m = [h_\infty/h_f(\gamma_\infty - 1)M_\infty^2]^{1/2}$$

which, for $M_\infty \approx 20$, is of the order of 10 for sharp bodies and varies from about 2 to 10 for blunt bodies. It is not clear whether θ_h or θ_m is the appropriate reference dimension; however, since the hypersonic wake is primarily an enthalpy excess wake,[10] θ_h seems preferable and will be adopted. Since physical lengths y are related to transformed lengths Y by approximately $y = (h_0/h_f)^{1/2}\,Y$, then, for the physical scale,

$$\Lambda/d = 0.37\,bM_\infty^{2/3}[(h_0/h_f)^{3/2}C_{D_f}x/d]^{1/3} \qquad (3)$$

Sample Calculations

In Fig. 1, relative velocity fluctuation and gas density fluctuation levels are plotted as computed from the given relations for two typical cases of interest, a 2-ft-diam 12° cone at $M_\infty = 22$ and 3 mm pressure and a $\frac{1}{4}$-in. sphere at $M_\infty = 8.5$ and 760 mm pressure. The mean values were computed as in Refs. 10 and 13. A value for a of $\frac{1}{5}$ was chosen; this is approximately the value measured by Townsend for velocity fluctuations in the self-preserving incompressible wake behind a cylinder. An interesting feature of these plots is the nonmonotonic change with distance; this result stems from the effect of "drag swallowing" by the wake. After the wake reaches the streamline corresponding to the bow shock-shoulder expansion intersection in the case of the cone, the wake edge enthalpy is no longer approximately constant but decreases rapidly for some distance. Thus, a local peak in the density fluctuation results. For the sphere, a nonmonotonic variation in density fluctuation also occurs; in this case it results from a rapid but continuously decreasing edge enthalpy.

In Fig. 2, the scale Λ from Eq. (3) is compared with the calculated wake width. For the lateral scale Λ_r, Townsend's measurements suggest $b_r \approx 0.05$, and this value is used in Fig. 2. The effect of a variable C_{D_f} is again apparent near $x/d \approx 500$, with the scale for the slender body increasing at a more rapid rate in this region, as does the wake width. Note that the magnitudes of the scales appear quite reasonable in comparison with the wake widths when Townsend's incompressible two-dimensional constant b is used.

At large x/d, with all the drag contained in the wake,

$$\Lambda/d \sim (C_D M_\infty^2)^{1/3}(x/d)^{1/3}$$

$$(\rho'/\rho)_0 \sim (C_D M_\infty^6)^{1/3}(x/d)^{-2/3}$$

For the two examples given, $C_D M_\infty^2$ is about equal, and hence the scales Λ/d should be about the same at large x/d. However, the $(C_D M_\infty^6)^{1/3}$ scaling for the fluctuations $(\rho'/\rho)_0$ indicates a value for the sphere about three times smaller than

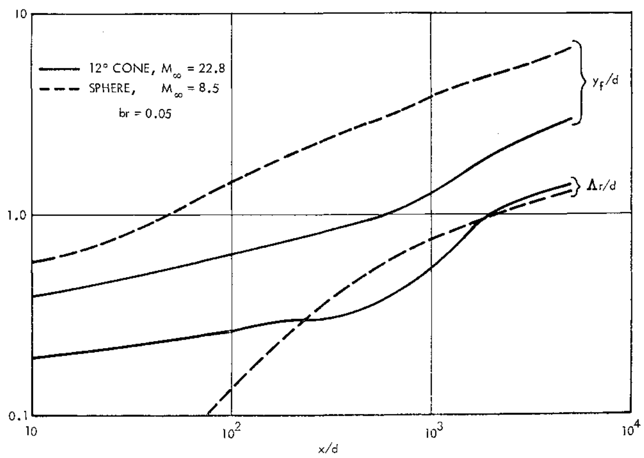

Fig. 2 Self-preserving integral scale for slender cone and sphere wakes.

¶ The linear relations are probably adequate for relative fluctuations smaller than about 0.4.[8]

** The stagnation enthalpy variation is of order U'/U and may be neglected. For $U'/h^{1/2} \ll 1$ the pressure fluctuations are are small; for a slender body or at large x, the level of dissociation is also small.

for the cone at large x/d. The results shown in Figs. 1 and 2 illustrate both of these conclusions.

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A Transient Solution of the Fokker-Planck Equation

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Introduction

In many cases of practical interest, it is found that the mean temperature of the electrons in an ionized gas is different from that of the gas molecules. Such cases exist, for example, when an initially hot electron gas is injected into a relatively cold neutral gas, or when the neutral molecule temperature and density are changing. The electrons do not follow these changes immediately, as many collisions of an electron with the neutral molecules are required before the electrons come into thermal equilibrium. In what follows, it is shown that a complete solution of the distribution function is forthcoming when the collision frequency is velocity independent, corresponding to the Maxwell law of interaction.

Fokker-Planck Equation

The Boltzmann equation is usually written as

$$(\partial f/\partial t) + \nabla f \cdot \mathbf{v} + \nabla_v f \cdot \mathbf{g} = C \tag{1}$$

where

f = velocity distribution function
t = time
C = collision integral

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and $\nabla f \cdot \mathbf{v}$ and $\nabla_v f \cdot \mathbf{g}$ represent the influences of diffusion and external forces, respectively.

The collision integral is given by Allis[1] as

$$C = \frac{m}{Mv^2}\frac{d}{dv}\left[v^2\nu\left(vf + \frac{kT}{m}\frac{df}{dv}\right)\right] \tag{2}$$

where

m = electron mass
M = molecular mass
v = electron velocity
ν = collision frequency
k = Boltzmann's constant
T = gas temperature

Equation (2) is actually the first term of the isotropic part of the collision integral for elastic collisions of electrons and molecules expanded in powers of m/M and is therefore a good approximation for the case to be considered here.

For a spatially uniform gas in which no external forces are acting, the Boltzmann equation is then written as

$$\frac{\partial f}{\partial t} = \frac{m}{Mv^2}\frac{\partial}{\partial v}\left[v^2\nu\left(vf + \frac{kT}{m}\frac{\partial f}{\partial v}\right)\right] \tag{3}$$

which is the usual form of the Fokker-Planck equation.

General Solution of the Fokker-Planck Equation

The collision frequency is, in general, a function of velocity, and it is expressed here as a general power series expansion in v, i.e.,

$$\nu = \Sigma \nu_n v^n \tag{4}$$

where the values of n may be positive and negative.

The Maxwellian distribution is written as

$$f = (A/\pi)^{3/2}e^{-Av^2} \tag{5}$$

where A is a function of time only. Substituting Eqs. (4) and (5) into Eq. (3) gives the following differential equation for the function A:

$$\frac{M}{m}\left(\frac{3}{2} - v^2A\right)\frac{dA}{dt} = A\Sigma\nu_n v^n \times \left\{n + 3 - 2A\left[\frac{kT}{m}(n+3) + v^2\right] + \frac{4kT}{m}A^2v^2\right\} \tag{6}$$

An examination of Eq. (6) reveals that, since A is independent of the velocity, the *only* nontrivial, consistent equation results when the collision frequency is velocity independent, i.e., when $\nu = \nu_0$, corresponding to the Maxwell law of interaction. The resulting equation for this case is

$$\frac{M}{m\nu}\frac{dA}{dt} + \frac{4kT}{m}A^2 - 2A = 0 \tag{7}$$

In general, both ν and T have general time variations. Consider the homogeneous equation

$$(M/m)\,(dA/d\tau) = 2A \tag{8}$$

where $d\tau = \nu dt$. The solution of Eq. (8) is just

$$A = e^{(2m/M)\tau} \tag{9}$$

The solution of the nonhomogeneous equation (7) is assumed to be of the form

$$A = Be^{(2m/M)\tau} \tag{10}$$

Substitution of Eq. (10) into Eq. (7) results in the following equation for B:

$$\frac{M}{m\nu}\frac{dB}{dt} = -\frac{4kT}{m}B^2e^{(2m/M)\tau} \tag{11}$$